NAG Toolbox for MATLAB

g01ee

1 Purpose

g01ee computes the upper and lower tail probabilities and the probability density function of the beta distribution with parameters a and b.

2 Syntax

```
[p, q, pdf, ifail] = g01ee(x, a, b, tol)
```

3 Description

The probability density function of the beta distribution with parameters a and b is:

$$f(B : a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} B^{a-1}(1-B)^{b-1}, \qquad 0 \leq B \leq 1; a, b > 0.$$

The lower tail probability, $\mathbf{p}(\mathbf{b} \leq \beta : a, b)$ is defined by

$$P(B \leq \beta : a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^{\beta} B^{a-1}(1-B)^{b-1}\, dB = I_{\beta}(a, b), \qquad 0 \leq \beta \leq 1; a, b > 0.$$

The function $I_x(a, b)$ is also known as the incomplete beta function.

The method used is similar to that described by Majumder and Bhattacharjee 1973, and uses the following three relations for the incomplete beta function (see Abramowitz and Stegun 1972):

$$I_x(a, b) = \frac{\Gamma(a+b)}{\Gamma(a+1)\Gamma(b)} x^a (1-x)^{b-1} + I_x(a+1, b-1) \tag{1}$$

$$I_x(a, b) = \frac{\Gamma(a+b)}{\Gamma(a+1)\Gamma(b)} x^a (1-x)^{b} + I_x(a+1, b) \tag{2}$$

$$I_x(a, b) = 1 - I_{1-x}(b, a) \tag{3}$$

If a is less than $(a+b)x$, then a and b are interchanged and $(1-x)$ replaces x, with relation (3) being used to obtain the final result.

Relation (1) is applied repeatedly until the second parameter is reduced to b', where $0 < b' \leq 1$. This produces a power series of finite length, in $x/(1-x)$, whose sum is found. If $b' = 1$, this sum equals $I_x(a, b)$, since $I_x(c, 1) = x^c/c$ for all $c > 0$.

Otherwise $\left(b' \neq 1\right)$, the integral $I_x(c, d)$ remains to be evaluated, where

$$\begin{gathered} c = a + b - b' \\ d = b' \\ 0 < b' < 1. \end{gathered}$$

Relation (2) applied repeatedly gives a convergent power series in x of infinite length.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

Majumder K L and Bhattacharjee G P 1973 Algorithm AS63. The incomplete beta integral *Appl. Statist.* **22** 409–411

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

β, the value of the beta variate.

Constraint: $0.0 \leq \mathbf{x} \leq 1.0$.

2: **a – double scalar**

a, the first parameter of the required beta distribution.

Constraint: $0.0 < \mathbf{a} \leq 10^6$.

3: **b – double scalar**

b, the second parameter of the required beta distribution.

Constraint: $0.0 < \mathbf{b} \leq 10^6$.

4: **tol – double scalar**

The relative accuracy required by you in the results. If g01ee is entered with **tol** greater than or equal to 1.0 or less than 10 times the ***machine precision*** (see x02aj), then the value of $10 \times$ ***machine precision*** is used instead.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **p – double scalar**

The lower tail probability, $\mathbf{p}(\mathbf{b} \leq \beta : a, b)$.

2: **q – double scalar**

The upper tail probability, $\mathbf{p}(\mathbf{b} \geq \beta : a, b)$.

3: **pdf – double scalar**

The probability density function, $f(\mathbf{b} : a, b)$.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01ee may return useful information for one or more of the following detected errors or warnings.

ifail $= 1$

On entry, $\mathbf{x} < 0.0$,
or $\mathbf{x} > 1.0$.

ifail $= 2$

On entry, $\mathbf{a} \leq 0.0$,
or $\mathbf{a} > 10^6$,
or $\mathbf{b} \leq 0.0$,
or $\mathbf{b} > 10^6$.

ifail $= 3$

The requested accuracy has not been achieved, see Section 7. Try using a larger value of **tol**. The values returned for **p** and **q** should be reasonable approximations.

ifail $= 4$

x is too far out into the tails for the probability to be evaluated exactly. The results returned are 0 and 1 as appropriate. These should be a good approximation to the required solution.

7 Accuracy

The convergence of series (2) is assumed when an upper bound on the sum of the remaining terms is less than **tol**. Summation also ceases if the relative change in the sum of the series is less than ***machine precision***, in which case full accuracy cannot be guaranteed.

The accuracy is limited by the error in evaluating the logarithm of the gamma function.

8 Further Comments

The time taken by g01ee will depend on the shape of the distribution. For highly skewed distributions with one of the values of a, b large and the other small, series (2) will take longer to converge than for distributions which are more symmetric.

9 Example

```
x = 0.25;
a = 1;
b = 2;
tol = 1.9;
[p, q, pdf, ifail] = g01ee(x, a, b, 'tol', tol)
```

```
p =
    0.4375
q =
    0.5625
pdf =
    1.5000
ifail =
           0
```